This page explains the logic of binomial option pricing models how option price is calculated from the inputs using binomial trees, and how these trees are built.All models simplify reality, in order to make calculations possible, because the real world (even a simple thing like stock price movement) is often too complex to describe with mathematical formulas.Binomial option pricing models make the following assumptions.Discrete StepsPrices don't move continuously (as Black-Scholes model assumes), but in a series of discrete steps.Time between steps is constant and easy to calculate as time to expiration divided by the model's number of steps.For example, if you want to price an option with 20 days to expiration with a 5-step binomial model, the duration of each step is 20/5 = 4 days. Once every 4 days, price makes a move.Up and Down MovesAt each step, the price can only do two things (hence binomial): Go up or go down.The sizes of these up and down moves are constant (percentage-wise) throughout all steps, but the up move size can differ from the down move size.For instance, at each step the price can either increase by 1.8% or decrease by 1.5%. These exact move sizes are calculated from the inputs, such as interest rate and volatility.Like sizes, the probabilities of up and down moves are the same in all steps. They must sum up to 1 (or 100%), but they don't have to be 50/50. Like sizes, they are calculated from the inputs.For example, from a particular set of inputs you can calculate that at each step, the price has 48% probability of going up 1.8% and 52% probability of going down 1.5%.This is all you need for building binomial trees and calculating option price.How to Calculate Option PriceThese are the things to do (not using the word steps, to avoid confusion) to calculate option price with a binomial model:Know your inputs (underlying price, strike price, volatility etc.).From the inputs, calculate up and down move sizes and probabilities.Build underlying price tree from now to expiration, using the up and down move sizes.The final step in the underlying price tree shows different underlying prices at expiration for different scenarios.From the above, calculate option payoff at expiration for different scenarios = the final step in the option price tree.Build the option price tree backwards from expiration to now.The price at the beginning of the option price tree is the current option price.Underlying Price TreeWe have already explained the logic of points 1-2. Exact formulas for move sizes and probabilities differ between individual models (for details see Cox-Ross-Rubinstein, Jarrow-Rudd, Leisen-Reimer). The rest is the same for all models.For now, let's use some round values to explain how binomial trees work:Up and down move sizes +1% and -1%Probabilities 50% eachCurrent underlying stock price $100The simplest possible binomial model has only one step. A one-step underlying price tree with our parameters looks like this:It starts with current underlying price (100.00) on the left. From there price can go either up 1% (to 101.00) or down 1% (to 99.00).There is no theoretical upper limit on the number of steps a binomial model can have. Generally, more steps means greater precision, but also more calculations. In this tutorial we will use a 7-step model.Binomial Tree CharacteristicsIndividual steps are in columns.The first column, which we can call step 0, is current underlying price.In each successive step, the number of possible prices (nodes in the tree), increases by one.The number of nodes in the final step (the number of possible underlying prices at expiration) equals number of steps + 1.There are two possible moves from each node to the next step up or down.There are also two possible moves coming into each node from the preceding step (up from a lower price or down from a higher price), except nodes on the edges, which have only one move coming in.Calculating the TreeKnowing the current underlying price (the initial node) and up and down move sizes, we can calculate the entire tree from left to right.Each node can be calculated either by multiplying the preceding lower node by up move size (e.g. by 1.02 if up move is +2%), or by multiplying the preceding higher node by down move size. Both should give the same result, because a b = b a.Paths and ProbabilitiesThere can be many different paths from the current underlying price to a particular node. For instance, up-up-down (green), up-down-up (red), down-up-up (blue) all result in the same price, and the same node.Notice how the nodes around the (vertical) middle of the tree have many possible paths coming in, while the nodes on the edges only have a single path (all ups or all downs). This reflects reality it is more likely for price to stay the same or move only a little than to move by an extremely large amount.If you are thinking of a bell curve, you are right. With growing number of steps, number of paths to individual nodes approaches the familiar bell curve.Option Payoff at ExpirationThe last step in the underlying price tree gives us all the possible underlying prices at expiration. For each of them, we can easily calculate option payoff the option's value at expiration.Two things can happen at expiration.If the option ends up in the money, we exercise it and gain the difference between underlying price S and strike price K:From a call, we gain S K.From a put we gain K S.If the above differences (potential gains from exercising) are negative, we choose not to exercise and just let the option expire. The option's value is zero in such case.Therefore, option value at expiration is:C = max(0, S K)P = max(0, K S)These option values, calculated for each node from the last column of the underlying price tree, are in fact the option prices in the last column of the option price tree.Option Price TreeWhile underlying price tree is calculated from left to right, option price tree is calculated backwards from the set of payoffs at expiration, which we have just calculated, to current option price.Each node in the option price tree is calculated from the two nodes to the right from it (the node one move up and the node one move down).We already know the option prices in both these nodes (because we are calculating the tree right to left).We also know the probabilities of each (the up and down move probabilities).With all that, we can calculate the option price as weighted average, using the probabilities as weights:... where Ou and Od are option prices at next step after up and down move, andp is probability of up move (therefore 1 p must be probability of down move).But we are not done. We must discount the result to account for time value of money, because the above expression is expected option value at next step, but we want its present value, one step earlier. The discount factor is:... where r is the risk-free interest rate and t is duration of one step in years, calculated as t/n, where t is time to expiration in years (days to expiration / 365), and n is number of steps.The formula for option price in each node (same for calls and puts) is:Using this formula, we can calculate option prices in all nodes going right to left from expiration to the first node of the tree which is the current option price, the ultimate output.American Options and Early ExerciseThe above formula holds for European options, which can be exercised only at expiration. This is why I have used the letter E, as European option or expected value if we hold the option until next step.American options can be exercised early. We must check at each node whether it is profitable to exercise, and adjust option price accordingly.American option price will be the greater of:What we would gain from exercisingThe option's expected value when not exercising = EWe need to compare the option price E with the option's intrinsic value, which is calculated exactly the same way as payoff at expiration:C = max(0, S K)P = max(0, K S)... where S is the underlying price tree node whose location is the same as the node in the option price tree which we are calculating.If intrinsic value is higher than E, the option should be exercised. Option price equals the intrinsic value.Otherwise (it is not profitable to exercise, so we keep holding the option) option price equals E.This is probably the hardest part of binomial option pricing models, but it is the logic that is hard the mathematics is quite simple.When implementing this in Excel, it means combining some IFs and MAXes:IF the option is American, option price is MAX of intrinsic value and E.Otherwise (it's European) option price is E.IF the option is a call, intrinsic value is MAX of zero and S K.Otherwise (it's a put) intrinsic value is MAX of zero and K S.We will create both binomial trees in Excel in the next part.Event: Lehman Brothers filed for bankruptcy on September 15, 2008the largest in U.S. history.Role in OTC Market: Heavily involved in high-risk derivatives.Cause: Inability to refinance short-term debt.At the time of its bankruptcy, Lehman Brothers had an extensive network of transactions, with hundreds of thousands outstanding across approximately 8,000 counterparties. The process of unwinding these transactions has posed significant challenges for both the Lehman liquidators and the involved counterparties, illustrating the complex and interconnected nature of modern financial markets.Portfolio:Value = $500,000\(\beta\) = 2.0Index = 1,000Target insured value = $450,000Risk-free rate = 12% p.a., dividend yield = 4%Contracts Required:\[\frac{500{,}000}{100 \times 1{,}000} \times 2 = 10\] put optionsIndex rises to 1,040 in 3 months: 4% returnTotal return: 5% (4% price + 1% dividends)Excess return: \(5\% - 3\% = 2\%\) (quarterly risk-free rate)Portfolio excess return: \(2\% \times 2 = 4\%\)Net return: \(4\% + 3\% - 1\% = 6\%\)Projected portfolio value:\[500{,}000 \times 1.06 = 530{,}000\]Result: Similar calculations can be carried out for other values of the index at the end of the three months. Appropriate strike price for the 10 put option contracts that are purchased is 960 (or 955 when we include dividends).Index in 3 MonthsPortfolio Value in 3 Months ($)1,080570,0001,040530,0001,000490,000960450,000920410,000880370,000 When I first encountered option pricing theory, I found it both fascinating and intimidating. The idea that we could mathematically model the value of financial derivatives seemed almost magical. Over time, I realized that the key to understanding this lies in breaking down the concepts into manageable pieces. One of the most intuitive and widely used models in this field is the Binomial Model. In this article, Ill walk you through the Binomial Model, its assumptions, applications, and how it compares to other pricing models. By the end, youll have a solid grasp of how this model works and why its so important in finance. Option pricing theory is a framework used to determine the fair value of financial options. An option is a contract that gives the buyer the right, but not the obligation, to buy or sell an underlying asset at a predetermined price (the strike price) on or before a specific date (the expiration date). The two main types of options are call options (which give the right to buy) and put options (which give the right to sell). Pricing options is tricky because their value depends on several factors, including the price of the underlying asset, volatility, time to expiration, and interest rates. Over the years, various models have been developed to tackle this problem, with the Black-Scholes Model and the Binomial Model being the most prominent. The Binomial Model, developed by Cox, Ross, and Rubinstein in 1979, is a discrete-time model for pricing options. Unlike the Black-Scholes Model, which assumes continuous time, the Binomial Model breaks down the time to expiration into a series of discrete intervals or steps. At each step, the price of the underlying asset can move up or down by a specific factor, creating a binomial tree of possible price paths. The beauty of the Binomial Model lies in its simplicity and flexibility. It can handle a wide range of scenarios, including American options (which can be exercised at any time before expiration) and options on assets that pay dividends. Before diving into the mechanics of the model, its important to understand its underlying assumptions: Discrete Time Intervals: The model divides the time to expiration into a finite number of intervals. Two Possible Movements: At each step, the price of the underlying asset can either move up or down by a fixed factor. No Arbitrage: The model assumes that there are no arbitrage opportunities, meaning its impossible to make a risk-free profit. Risk-Neutral Valuation: The model operates under the assumption that investors are risk-neutral, meaning they are indifferent to risk when pricing options. Lets start by constructing a simple binomial tree. Suppose we have a stock currently priced at S_0. Over a small time interval \Delta t, the stock price can either move up to S_0 \times u or down to S_0 \times d, where u and d are the up and down factors, respectively. The up and down factors are calculated using the following formulas: u = e^{\sigma \sqrt{\Delta t}} d = e^{-\sigma \sqrt{\Delta t}} Here, \sigma is the volatility of the underlying asset, and \Delta t is the length of each time step. Lets say we have a stock priced at $100, with a volatility of 20% and a time step of 1 year. Using the formulas above, we can calculate the up and down factors: u = e^{0.2 \times \sqrt{1}} = e^{0.2} \approx 1.2214 d = e^{-0.2 \times \sqrt{1}} = e^{-0.2} \approx 0.8187 So, after one year, the stock price can either move up to: S_u = 100 \times 1.2214 = 122.14 or down to: S_d = 100 \times 0.8187 = 81.87 This creates a simple one-step binomial tree: S_u = 122.14 / S_0 = 100 \ S_d = 81.87 Now that weve built the binomial tree, lets use it to price a European call option. A European call option gives the holder the right to buy the underlying asset at the strike price K at expiration. The value of the option at expiration is straightforward: C_u = \max(S_u - K, 0) C_d = \max(S_d - K, 0) To find the options value at the present time, we need to work backward through the tree using the concept of risk-neutral probability. Under the risk-neutral valuation framework, the expected return of the underlying asset is the risk-free rate r. The risk-neutral probability p of an up movement is given by: p = \frac{e^{r \Delta t} - d}{u - d} Using this probability, we can calculate the present value of the option as the discounted expected payoff: C_0 = e^{-r \Delta t} (p C_u + (1 - p) C_d) Lets continue with our previous example. Suppose the strike price K is $100, the risk-free rate r is 5%, and the time to expiration is 1 year. First, calculate the risk-neutral probability: p = \frac{e^{0.05 \times 1} - 0.8187}{1.2214 - 0.8187} \approx \frac{1.0513 - 0.8187}{0.4027} \approx 0.576 Next, calculate the options payoff at expiration: C_u = \max(122.14 - 100, 0) = 22.14 C_d = \max(81.87 - 100, 0) = 0 Finally, calculate the present value of the option: C_0 = e^{-0.05 \times 1} (0.576 \times 22.14 + (1 - 0.576) \times 0) \approx 0.9512 \times 12.76 \approx 12.14 So, the fair value of the European call option is approximately $12.14. While the one-step binomial tree is useful for illustration, real-world applications often require multiple steps to capture the complexity of price movements. The process remains the same, but the tree grows exponentially with each additional step. Lets extend our previous example to a two-step binomial tree. Each step is 6 months (\Delta t = 0.5 years), and the total time to expiration is 1 year. First, calculate the up and down factors: u = e^{0.2 \times \sqrt{0.5}} \approx 1.1519 d = e^{-0.2 \times \sqrt{0.5}} \approx 0.8681 Next, build the binomial tree: S_uu = 100 \times 1.1519 \times 1.1519 \approx 132.69 / S_u = 100 \times 1.1519 \approx 115.19 / \ S_0 = 100 S_ud = 100 \times 1.1519 \times 0.8681 \approx 100 \ / S_d = 100 \times 0.8681 \approx 86.81 \ S_dd = 100 \times 0.8681 \times 0.8681 \approx 75.36 Now, calculate the options payoff at expiration: C_{uu} = \max(132.69 - 100, 0) = 32.69 C_{ud} = \max(100 - 100, 0) = 0 C_{dd} = \max(75.36 - 100, 0) = 0 Next, calculate the risk-neutral probability for each step: p = \frac{e^{0.05 \times 0.5} - 0.8681}{1.1519 - 0.8681} \approx \frac{1.0253 - 0.8681}{0.2838} \approx 0.554 Finally, work backward through the tree to find the options present value: C_u = e^{-0.05 \times 0.5} (0.554 \times 32.69 + (1 - 0.554) \times 0) \approx 0.9753 \times 18.11 \approx 17.66 C_d = e^{-0.05 \times 0.5} (0.554 \times 0 + (1 - 0.554) \times 0) = 0 C_0 = e^{-0.05 \times 0.5} (0.554 \times 17.66 + (1 - 0.554) \times 0) \approx 0.9753 \times 9.78 \approx 9.54 So, the fair value of the European call option in this two-step model is approximately $9.54. Flexibility: The Binomial Model can handle a wide range of scenarios, including American options and options on dividend-paying stocks. Intuitive: The step-by-step approach makes it easier to understand and visualize. Discrete Time: Its particularly useful when dealing with assets that have discrete price movements or when continuous-time models are impractical.Computational Complexity: As the number of steps increases, the size of the binomial tree grows exponentially, making calculations more cumbersome. Approximation: The model is an approximation of reality, and its accuracy depends on the number of steps used. Assumptions: Like all models, the Binomial Model relies on assumptions that may not hold in real-world markets. The Black-Scholes Model is another widely used option pricing model. While both models aim to achieve the same goal, they differ in their approach and assumptions. FeatureBinomial ModelBlack-Scholes ModelTimeDiscreteContinuousFlexibilityHandles American and exotic optionsPrimarily for European optionsComplexityEasier to understandMore mathematically complexComputational LoadIncreases with more stepsLess computationally intensive In practice, the Binomial Model is often used as a stepping stone to understanding the Black-Scholes Model. While the Black-Scholes Model is more elegant and computationally efficient, the Binomial Models flexibility makes it a valuable tool in many situations. The Binomial Model is widely used in finance for pricing options, risk management, and strategic decision-making. Here are a few examples: Pricing American Options: Since American options can be exercised at any time before expiration, the Binomial Models discrete-time approach is particularly useful. Dividend-Paying Stocks: The model can easily incorporate dividends, making it suitable for pricing options on dividend-paying stocks. Employee Stock Options: Companies often use the Binomial Model to value employee stock options, which typically have unique features like vesting periods. The Binomial Model is a powerful and intuitive tool for pricing options. While it has its limitations, its flexibility and simplicity make it a cornerstone of option pricing theory. By breaking down the time to expiration into discrete steps, the model allows us to capture the complexity of price movements and calculate the fair value of options with precision. BLMKE, Andreas. How to invest in structured products: a guide for investors and investment advisors. Chichester: Wiley, 2009. xvi, 374. ISBN 9780470746790.Learning Outcomes:Identify the key characteristics and benefits of structured products as investment vehicles.Describe the role and impact of the European Structured Investment Products Association (EUSIPA) in the structured products market.Differentiate between various types of structured products, including investment certificates and barrier options.Evaluate the mechanisms and behavior of capital protection and participation investment products through theoretical and case study approaches. Structured products enjoy significant popularity across European states, primarily due to their capacity to balance risk and reward. These financial instruments offer several compelling advantages:Capital Protection and Market Participation: Investors are drawn to structured products because they provide a mechanism to protect all or part of the invested capital while still allowing participation in the gains of the capital markets.Accessibility to Regional Markets: These products facilitate access to regional markets and asset classes that may otherwise be inaccessible through direct investments.Diverse Return Profiles: Structured products can be designed with a wide array of return profiles, adapting to various investor needs and market conditions. They remain effective across different market movementsproviding potential returns in rising, sideways, or falling markets.Liquidity: Market makers enhance the liquidity of structured products by continuously buying and selling them, which ensures a stable market presence and availability.Sophisticated Investment Strategies: Investors can implement complex investment strategies typically available to advanced traders through the acquisition of a single structured product.Regulatory Oversight and Transparency: These products are often listed on official, regulated markets, adding a layer of security and transparency for investors.Tax Advantages: In some jurisdictions, structured products offer favorable tax conditions, enhancing their attractiveness as investment options.Official Website: EUSIPAMarket Insights: EUSIPA Market ReportsFounded in 2009, EUSIPA represents a collective of national issuer associations from multiple European countries including Austria, France, Germany, Italy, Sweden, Belgium, the UK, Switzerland, and The Netherlands. As an international non-profit association governed under Belgian law, EUSIPA also maintains a presence in the EU transparency register.EUSIPA aims to foster transparency and establish uniform market standards across Europe. It serves as a pivotal platform for its members to engage in meaningful dialogue with European policymakers, ensuring that the voices of issuers are heard in the regulatory landscape.Definition by Andreas BlmkeStructured products are financial assets, which consist of various elemental components, combined to generate a specific risk-return profile (not replicable with stocks and bonds) adapted to an investors needs.Structured products, often referred to as investment certificates, are essentially securitized derivatives. These are complex financial contracts encapsulated within a single security that trades on exchanges similar to stocks. These instruments are crafted and issued by financial institutions and are utilized by both retail and institutional investors. They can be traded on stock exchanges or dealt directly between parties in over-the-counter (OTC) transactions. Structured products carry inherent credit risks as they are issued in the form of bearer bonds. This means the issuers entire assets back the liability on these products. The quality and safety of structured products are intrinsically linked to the creditworthiness of the issuer. Like traditional bonds, these products are exposed to issuer risk, which implies that in the event of issuer bankruptcy, both bonds and structured products are treated as part of the bankruptcy estate.To mitigate such risks, investors are advised to:Opt for products issued by financially robust institutions.Diversify their investments across various issuers.Continuously monitor the financial health of the issuers over time. Structured products do not follow a universal standard for categorization. However, common classifications are often derived from industry associations such as:EUSIPA: European Structured Investment Products Association provides a framework for categorizing structured products within Europe.View the EUSIPA Derivatives Map for details.SVSP: Swiss Structured Products Association offers an alternative categorization scheme.Explore the SVSP Derivative Map for comparison.Classical structured products representation Structured products attract both private and institutional investors when traditional investment avenues do not meet their specific needs. These needs might include the desire for returns higher than the risk-free rate while still benefiting from capital protection. Structured products are particularly appealing in scenarios where conventional investments either do not provide sufficient returns or fail to address specific financial goals and risk profiles. Sellers of structured products, typically financial institutions, are driven by profit. They utilize sophisticated mathematical models to determine a fair value of the product at issuance. This fair value is then increased by a spread which covers various costs associated with the product over its lifetime. These costs include but are not limited to:Secondary market activitiesListing feesProduction of term-sheetsSettlement processesAmong these, the most significant cost factor is hedging. Hedging expenses are challenging to predict in advance as they depend on market dynamics over the products life. The profit from structured products, however, is only realized at their expiry, and having a large and diverse portfolio helps in more effectively hedging as some products can offset the risks of others. Structured products can be accessed and traded through various platforms, which serve as marketplaces for these financial instruments:Deutsche Bank X-markets: Offers a wide range of structured products for different investment strategies.Website: Deutsche Bank X-marketsBrse Stuttgart: Known for its user-friendly approach to trading structured products among other securities.Brse Frankfurt: One of Europes largest trading centers for securities, including derivatives and structured products. For further research and detailed insights into the market for structured products, the following resources are invaluable:European Structured Investment Products Association (EUSIPA): WebsiteGerman Derivatives Association: WebsiteSwiss Structured Products Association (SSPA): WebsiteUK Structured Products Association: WebsiteItalian Association of Certificates and Investment Products (ACEPI): Website Barrier options are exotic call or put options that include a barrier condition placed above or below the strike that, when crossed, either transfroms the exotic option into a plain vanilla option (in barriers) or cancels it altogether (out barriers).Barrier options are integrated into various structured financial products such as barrier reverse convertibles and bonus certificates. Recognized for their complexity, these options introduce a conditional component to the standard option mechanism, making the final payoff uncertain until the options maturity. Barrier options can be classified into four main types based on the direction of the barrier and its effect:Up & Out: The option becomes void if the underlying assets price goes above the barrier.Up & In: The option comes into existence when the underlying assets price goes above the barrier.Down & Out: The option becomes void if the underlying assets price falls below the barrier.Down & In: The option comes into existence when the underlying assets price falls below the barrier.Additionally, these options can feature a rebate, a predefined amount paid to the option holder if the barrier is breached before maturity. Barrier options are generally more cost-effective than their plain vanilla counterparts due to the added condition of the barrier. The pricing dynamics vary significantly between in and out options:For In Options: As the maturity increases, the price approaches that of a plain vanilla option, especially as the price of the underlying asset approaches the barrier.For Out Options: Longer maturities reduce the price, potentially approaching zero as the assets price nears the barrier. Structured products typically incorporate barrier options with specific monitoring styles:American-style Barriers: These allow the barrier condition to be triggered at any point during the options life, including intraday events, and are known for continuous monitoring.European-style Barriers: These restrict the barrier condition to only be checked at the maturity of the option.Although most structured products utilize American-style barriers due to cost-effectiveness, the investor focus generally lies elsewhere rather than on the barrier type itself.ExampleAmerican Barrier (Original)European BarrierShark NoteBarrier 131.5%, Rebate 7.5%Barrier 123%, Rebate 7.5%Barrier Reverse ConvertibleCoupon 10.4%, Barrier 75%Coupon 8.6%, Barrier 75%Bonus CertificateBonus 9%, Barrier 65%Bonus 2.5%, Barrier 65%Window-style Barriers: Situated between American and European styles, window barrier options are designed to activate or deactivate only during specific periods within the products life. For example, the barrier in some reverse convertibles might only be relevant during the final three months of a one-year term. Although not common, window barriers provide an opportunity for investors to avoid premature knockouts, with the value difference between American and window options typically being minimal. Capital guaranteed products ensure the redemption of the initial capital invested at maturity, while also allowing participation to varying degrees in the performance of an underlying risky asset.These products are distinguished by three primary features:Limited Loss Potential: The potential loss is confined to the level of the capital guarantee, not accounting for the issuers credit risk.Participation in Underlying Assets: Investors gain exposure to the performance of selected assets.Minimal Guaranteed Income: Typically, these products offer low or no guaranteed income, focusing instead on capital preservation and growth through asset performance.Its crucial to consider the opportunity costs, such as foregone dividends or the risk-free rate. While attractive at first glance, the actual benefit depends significantly on the performance of the underlying asset at maturity:If the asset performs well, the capital guarantee becomes redundant.If the asset performs poorly, it might have been better not to invest.Capital guaranteed products serve as a prudent option for investors who prefer not to remain in cash but are also uncertain about future market directions.Classical construction of capital guaranteed products involves:Issuing a zero-coupon bond matching the maturity of the product to ensure capital return.Purchasing a call option on the underlying risky asset to allow for profit participation. If we consider an interest rate of 4% with a 5-year maturity: \[\text{Zero-bond Price} = \frac{100\%}{(1+4\%)^5} = 82.19\%\]The available funds to purchase options (the discount) is: \[\text{Discount} = 100\% - 82.19\% = 17.81\%\]Participation rate is calculated as: \[\text{Participation} = \frac{\text{Discount}}{\text{Option cost}}\] Two critical factors affect the desirability and effectiveness of capital guaranteed products:Interest Rates: Higher rates increase the discount, thereby enhancing the capacity to purchase options.Volatility of the Underlying: Lower volatility reduces option costs, improving participation rates.Options for enhancing attractiveness include reducing the capital guarantee below 100%, adding caps or exotic features like knock-out barriers, and utilizing out-of-the-money options. Exchangeable CertificatesMarket Expectations: Rising volatility, sharply rising or falling underlying.Minimum redemption at maturity equals the capital protection (e.g., 100% of nominal).Value may fall below capital protection during the products life.Unlimited upside above the strike price, with possible coupon payments.Capped Capital ProtectionMarket Expectations: Rising underlying, potential for sharp falls.Guaranteed minimum redemption at expiry.Participation in positive performance up to a specified cap.Limited profit potential due to the cap.Shark Note - Knock-OutShark Note ConstructionMarket Expectations: Rising underlying, unlikely to breach a set barrier.Full capital protection with a short- to medium-term horizon.Uses an up-and-out call option; if the barrier is touched, a rebate may be paid.Participation in performance until a barrier is hit; if breached, participation ends and a rebate might be paid.Redemption Scenarios:If the underlying is below 100% of its initial value at maturity: 100% capital guarantee.If above 100% without touching the barrier: 100% + participation.If the barrier is touched: 100% + rebate.Strategic Use:Set a high barrier to minimize knock-out risk or a high rebate to enhance returns if knock-out occurs.Autocall feature allows early redemption if the barrier is breached, suitable for reinvestment without waiting for product maturity.Capital Protection with CouponMarket Expectations: Rising underlying, potential sharp falls.Guaranteed capital protection at maturity.Periodic coupon payments linked to the performance of the underlying.Limited upside potential.Match the product with your investment horizon; the capital guarantee is effective only at maturity.Avoid products with lower capital guarantee than 90% (e.g., 90% guarantee means that underlying must perform by more than 10% to be break even, not considering any opportunity cost).Ensure the participation rate is at least 80%.Verify the issuers credit rating.Prefer shorter maturity periods for products like Shark Notes to reduce risk (no more than two or three years).CharacteristicDetailsUnderlying Risky AssetEurostoxx50 IndexMaturity4 yearsImplied Volatility23%Assets Dividend Yield (p.a.)4%Interest Rate Level (4-year swap rate, p.a.)4.5%Capital Guarantee Level100%Initial Participation100%Key Variables:Interest Rate: A primary determinant of the price of the zero-coupon bond component of the product.Volatility: Crucial for the valuation of the call option embedded in the product. High volatility increases the potential upside, impacting the options price more significantly when the asset is at-the-money.Spot price variations for classical capital guaranteed productsThis graph demonstrates the relationship between the spot price of the underlying asset and the price of the capital guaranteed product. The slope of the line, particularly when it approaches a 45-degree angle, indicates a delta of 100%, where the products price moves one-to-one with the underlying asset.Price as a function of maturityThe slope of this graph illustrates how the delta of the product changes with time. Typically, the products participation at inception ranges from 40%-60% of its maturity level, reflecting the initial risk profile and pricing.Note: Capital guarantee is contingent on purchasing the product at par and is valid only at maturity, not before. The guarantee level remains constant throughout the products life.Implied Volatility Fluctuations:An increase in implied volatility (e.g., from 23% to 33%) can enhance the value of the call option component, as shown in the corresponding graph.A decrease (e.g., from 23% to 13%) typically lowers the call options value due to reduced potential for high returns.Implied volatility increase (from 23% to 33%)Implied volatility decrease (from 23% to 13%)Volatility is more influential when the product is at-the-money and can mitigate some losses through increased option premiums during downturns in the assets price.Interest Rate Impact:Rising interest rates lead to lower prices for the zero-coupon bond component, influencing the overall valuation negatively, especially if the underlying assets price falls simultaneously.Conversely, falling interest rates increase the bonds value, cushioning any adverse effects from a drop in the underlying assets price.Interest rate increase (from 4.5% to 6.5%)Interest rate decrease (from 4.5% to 2.5%)Shifts in implied volatility influence the capital guaranteed product most when at-the-money.An increase in volatility tends to increase the value of the product.A positive shift in interest rates lowers the value of the product most when the embedded call is out-of-the-money.When the call is deep in-the-money, an interest rate shift has less impact.Everything else held equal, the passing of time (without movement on the spot) is positive for the value of the product over time, since the time value lost on the call is more than offset by the gain in the zero bond value. Participation products are investment vehicles that link returns directly to the performance of their underlying assets, sometimes featuring conditional downside protection or a leveraged upside.Key Characteristics:Risk Profile: Generally higher risk compared to capital protection and yield enhancement products due to the absence of capital guarantees.Underlying Assets: Typically stocks or stock indices, but can also include commodities, real estate, and more exotic assets.Liquidity and Efficiency: Often very liquid, these products compete directly with ETFs in providing exposure to specific markets, themes, or regions.Function: Mirrors the performance of one or more underlying assets. Commonly tracks excess returns (excluding dividends/yields).Structure: Comprised of a zero-strike call option, which values the asset minus any discounted dividends or yields.Advantages:Access to Difficult Markets: Allows investment in markets or assets that are otherwise inaccessible via traditional instruments, such as private equity or certain commodities.Tax Efficiency: May offer favorable tax treatments compared to direct investments in the underlying assets.Function: Combines the features of a tracker certificate with conditional downside protection.Structure: Includes a zero-strike call option and a long down-and-out put option.Key Parameters:Participation Level: Degree to which the investor gains from positive performance of the underlying.Bonus Level: Additional return offered if the underlying performs above a certain threshold without breaching a downside barrier.Barrier Level: The price level below which the downside protection is activated.Maturity: Typically short, recommended no longer than two to three years.Function: Provides positive participation in both the upside and downside movements of the underlying asset.Structure: Consists of a long zero-strike call and double down-and-out put options.Function: Designed for aggressive investment strategies, providing enhanced returns if the underlying outperforms expected dividends.Structure: Combines a zero-strike call with multiple long at-the-money calls, funded by dividends.Volatility and Dividends: Optimal conditions include low volatility (for cheaper options) and high dividend yields.Function: Provides leveraged exposure to the underlying asset up to a capped level.Structure: A zero-strike call combined with a long at-the-money call and two short calls at higher strikes.Scenario Planning: Best utilized in a moderately bullish scenario with falling volatility.Timing and Maturity: Critical due to the short duration of the product, typically 3-9 months. Participation products such as bonus, turbo, airbag, and outperformance certificates are heavily influenced by factors like implied volatility, dividend yields, and interest rates. These factors shape the products performance and its strategic suitability for different market conditions.Factor (Increase)Impact on Products PriceEffect LevelSpot PricePositiveMaximumImplied VolatilityVariableHighImplied CorrelationPositiveMedium-LowInterest RatesNegativeLowDividendsNegativeMediumInitial Sensitivity: At issuance, the delta of a bonus certificate is approximately 1, meaning its price moves almost one-for-one with the underlying asset. However, this sensitivity decreases if the spot price approaches the barrier.Price Stability: Bonus certificates tend to underperform during market downturns due to the drop in market value, despite their protective features.Price as a function of maturityMaturity: Shorter maturities are preferable to reduce exposure to prolonged market volatility.Barrier Level: The barrier should be set considering the worst-case market scenario to ensure effective downside protection.Leverage and Sensitivity: As the spot price nears the barrier, the certificates delta becomes highly volatile, which can lead to significant price swings.Initial Conditions: High implied volatility at issuance allows for better pricing of protective options.Post-Issuance Volatility: A decrease in implied volatility post-issuance generally benefits the mark-to-market value of the certificate.Increase in implied volatility (from 23% to 33%)Decrease in implied volatility (from 23% to 13%)What is the value of the capital guarantee certificate at maturity if the value of the underlying asset ends at 120, the strike price of the certificate is 100, level of guarantee 100%, and participation 80%?What is the value of the capital guarantee certificate at maturity if the value of the underlying asset ends at 90, the strike price of the certificate is 100, level of guarantee 100%, and participation 80%?What is the value of the capital guarantee certificate at maturity if the value of the underlying asset ends at 63, the strike price of the certificate is 40, the cap is 50, level of guarantee 100%, and participation 130%?What is the value of the capital guarantee certificate with knock-out at maturity if the value of the underlying asset ends at 50, the strike price of the certificate is 55, the knock-out barrier is 67, level of guarantee 100%, and participation 110%? The highest value of the underlying asset during maturity was 70.What is the value of the capital guarantee certificate with knock-out at maturity if the value of the underlying asset ends at 63, the strike price of the certificate is 55, the knock-out barrier is 67, level of guarantee 100%, and participation 110%? The highest value of the underlying asset during maturity was 65.What is the value of the outperformance certificate at maturity if the value of the underlying asset ends at 56, the strike price of the certificate is 40, and the participation is 170%?What is the value of the outperformance certificate at maturity if the value of the underlying asset ends at 77, the strike price of the certificate is 40, the cap is 70, and the participation is 130%?What is the value of the bonus certificate at maturity if the value of the underlying asset ends at 35, the strike price of the certificate is 40, the knock-out barrier is 30? The lowest value of the underlying asset during maturity was 35.What is the value of the bonus certificate at maturity if the value of the underlying asset ends at 35, the strike price of the certificate is 40, the knock-out barrier is 30? The lowest value of the underlying asset during maturity was 25.What is the value of the twin-win certificate at maturity if the value of the underlying asset ends at 44, the strike price of the certificate is 50, the knock-out barrier is 40? The lowest value of the underlying asset during maturity was 35.What is the value of the twin-win certificate at maturity if the value of the underlying asset ends at 44, the strike price of the certificate is 50, the knock-out barrier is 40? The lowest value of the underlying asset during maturity was 41.

Numerical method for the valuation of financial optionsIn finance, the binomial options pricing model (BOPM) provides a generalizable numerical method for the valuation of options. Essentially, the model uses a "discrete-time" (lattice based) model of the varying price over time of the underlying financial instrument, addressing cases where the closed-form BlackScholes formula is wanting, which in general does not exist for the BOPM.[1]The binomial model was first proposed by William Sharpe in the 1978 edition of Investments (ISBN013504605X),[2] and formalized by Cox, Ross and Rubinstein in 1979[3] and by Rendleman and Bartter in that same year.[4]For binomial trees as applied to fixed income and interest rate derivatives see Lattice model (finance) Interest rate derivatives.The Binomial options pricing model approach has been widely used since it is able to handle a variety of conditions for which other models cannot easily be applied. This is largely because the BOPM is based on the description of an underlying instrument over a period of time rather than a single point. As a consequence, it is used to value American options that are exercisable at any time in a given interval as well as Bermudan options that are exercisable at specific instances of time. Being relatively simple, the model is readily implementable in computer software (including a spreadsheet).Although higher in computational complexity and computationally slower than the BlackScholes formula, it is more accurate, particularly for longer-dated options on securities with dividend payments. For these reasons, various versions of the binomial model are widely used by practitioners in the options markets.[citation needed]For options with several sources of uncertainty (e.g., real options) and for options with complicated features (e.g., Asian options), binomial methods are less practical due to several difficulties, and Monte Carlo option models are commonly used instead. When simulating a small number of time steps Monte Carlo simulation will be more computationally time-consuming than BOPM (cf. Monte Carlo methods in finance). However, the worst-case runtime of BOPM will be O(2n), where n is the number of time steps in the simulation. Monte Carlo simulations will generally have a polynomial time complexity, and will be faster for large numbers of simulation steps. Monte Carlo simulations are also less susceptible to sampling errors, since binomial techniques use discrete time units. This becomes more true the smaller the discrete units become.Binomial Lattice with CRR formulaefunction americanPut(T, S, K, r, sigma, q, n) { ' T... expiration time ' S... stock price ' K... strike price ' r... interest rate ' sigma... volatility of the stock price ' q... dividend yield ' n... height of the binomial tree deltaT:= T / n; up:= exp(sigma * sqrt(deltaT)); p0:= (up * exp(-q * deltaT) - exp(-r * deltaT)) / (up^2 - 1); p1:= exp(-r * deltaT) - p0; ' initial values at time T for i:= 0 to n { p[i]:= K - S * up^(2*i - n+1); if p[i] < 0 then p[i]:= 0; } ' move to earlier times for j:= n-1 down to 0 { for i:= 0 to j { ' binomial value p[i]:= p0 * p[i+1] + p1 * p[i]; ' exercise value exercise:= K - S * up^(2*i - j+1); if p[i] < exercise then p[i]:= exercise; } } return americanPut:= p[0];}The binomial pricing model traces the evolution of the option's key underlying variables in discrete-time. This is done by means of a binomial lattice (Tree), for a number of time steps between the valuation and expiration dates. Each node in the lattice represents a possible price of the underlying at a given point in time.Valuation is performed iteratively, starting at each of the final nodes (those that may be reached at the time of expiration), and then working backwards through the tree towards the first node (valuation date). The value computed at each stage is the value of the option at that point in time.Option valuation using this method is, as described, a three-step process:Price tree generation,Calculation of option value at each final node,Sequential calculation of the option value at each preceding node.The tree of prices is produced by working forward from valuation date to expiration.At each step, it is assumed that the underlying instrument will move up or down by a specific factor (u {\displaystyle u} or d {\displaystyle d}) per step of the tree (where, by definition, u 1 {\displaystyle u\geq 1} and 0 < d 1 {\displaystyle 0\epsilon } \Delta L_t, \quad \mu (\theta) =r+b(\epsilon)-\theta -q-\frac{\sigma ^2}{2}onumber \\ b(\epsilon)= & \int _{-\epsilon }^\epsilon xu (x)dx = \epsilon ^2 \int _0^1 x\{ u (\epsilon x) - u (-\epsilon x) \}dx\;. \end{aligned}$$ (A.2) To correctly define \(b(\epsilon)\), we make the following assumption: Assumption A.1 We assume that the integral (A.2) takes a finite value. This assumption always holds for compound Poisson processes. Moreover, Muroi and Suda (2023) show that it also holds for the CGMY and NIG models. [2] We approximate the small noise component, \(\{M_t=\sigma B_t + (L_t^{\epsilon } - b(\epsilon)t) \}\). We introduce a sequence of independent and identically distributed random variables \(\{\xi _{i} \}_{i=1}^N\) representing the small noise component as$$\begin{aligned} {\mathbb Q}\left[\xi _{i} \Delta x=-\Delta x \right] =p_{-1}, \ {\mathbb Q}\left[\xi _{i} \Delta x=0 \right] =p_{0},\ {\mathbb Q} \left[\xi _{i} \Delta x=\Delta x \right] =p_{1}\;. \end{aligned}$$We fit the first- and second-order moments to the random variable \(M_{t+\Delta t}-M_t\). Then, we have$$p_{\pm 1}=\frac{1}{2} \left(\frac{\sigma ^2+\sigma ^2(\epsilon)}{\eta ^2} \right),\quad p_{0}=1-p_{-1}-p_{1}=1-\left(\frac{\sigma ^2+\sigma ^2(\epsilon)}{\eta ^2} \right) \;,$$where \(\sigma ^2(\epsilon)\) is given by$$\sigma ^2(\epsilon):= \int _{-\epsilon }^\epsilon x^2u (x)dx - b(\epsilon)^2 = \epsilon ^3 \int _0^1 x^2\{ u (\epsilon x) + u (-\epsilon x) \}dx - b(\epsilon)^2\;.$$We must appropriately choose the parameter \(\eta\) in \(\Delta x=\eta \sqrt{\Delta t}\), such that \(\{p _{-1},p _0,p _1\}\) are probabilities: \(0 \le p _{j} \le 1,\ (j=0,\pm 1)\). [3] We approximate a large jump part, \(N_t\), driven by the compound Poisson process. We introduce new variables \(s_j\ (j=\pm 2,\pm 3,\cdots)\) defined as$$s_j = \int _{(j-1/2)\Delta x}^{(j+1/2)\Delta x}u (x)dx = \int _{-1}^1 u ((j+s/2)\Delta x)ds \frac{\Delta x}{2}.$$The intensity of the compound Poisson process \(N_t\) is given by \(\lambda =\sum _{j=\pm 2, \pm 3, \cdots } s_j\). We introduce a sequence of independent and identically distributed random variables \(\{\eta _{i} \}_{i=1}^N\) representing the large jump component. The probability distribution is given by \({\mathbb Q}\left[\eta _{i} \Delta x =\Delta x \right] = q_{j} \ (j=0, \pm 2, \pm 3, \cdots)\;,\) where \(\{q_{j}\}_j\) are given by$$\begin{aligned} q_{j} = & (1-e^{-\lambda \Delta t}) s_j / \lambda,\quad (j=\pm 2,\pm 3,\cdots) \\ q_{0}= & 1 - \sum _{j=\pm 2,\pm 3,\cdots }q_{j} = e^{-\lambda \Delta t}\;. \end{aligned}$$The probability of a jump occurring during a short period \(\Delta t\) is given by \((1-e^{-\lambda \Delta t})\). [4] Considering that the constant \(\theta\) satisfies the condition \({\mathbb E}\left[e^{L_t} \right] =e^{\theta t}\), we have \({\mathbb E}\left[e^{L_t+\sigma B_t} \right] ={\mathbb E}\left[e^{L_t} \right] {\mathbb E} \left[e^{\sigma B_t} \right] =e^{(\theta +\frac{\sigma ^2}{2}) t}\) using the independence of the two stochastic processes \(\{ L_t \}\) and \(\{B_t \}\). We obtain the following equation:$$\theta +\frac{\sigma ^2}{2} = \frac{1}{\Delta t} \log {\mathbb E} \left[e^{\sigma \Delta B_t+\Delta L_t} \right] \approx b(\epsilon) + \frac{1}{\Delta t}\log \left(\sum _{j=0,\pm 1}^{}e^{j\Delta x}{p}_{j}\right) + \frac{1}{\Delta t}\log \left(\sum _{j=0,\pm 2,\pm 3,\cdots }e^{j\Delta x}q_{j} \right) \;.$$We can numerically calculate \(\theta\) using this equation. [5] We compute the following characteristic function for \(Z=\xi _i+\eta _i\):$$\phi (\omega):= \left(p_0+ \sum _{m=-L, m e -1,0,1}^{M} e^{i \omega m}p_m \right) ^N \left(e^{-i \omega }q_{-1}+q_0+e^{i \omega }q_1 \right) ^N\;.$$[6] Given that the drift term is not included in the lattice, we must modify the discrete cosine coefficients for the call and put options at the maturity date given in Eq.2.6 as$$\begin{aligned} V_k^{J-1}= \left\{ \begin{array}{l} S e^{\mu (\theta)T} \chi _k(\tilde{\rho } +1,b)-K \psi _k(\tilde{\rho } +1,b), \ (call)\\ K \psi _k(a,\tilde{\rho })-S e^{\mu (\theta)T} \chi _k(a,\tilde{\rho }), \ (put), \end{array} \right. \end{aligned}$$where \(\tilde{\rho }\) is given as \(\tilde{\rho } = \left\lfloor \frac{\log (K/Se^{\mu (\theta)T})}{\sigma \sqrt{\Delta t}} \right\rfloor\). We also need to change the payoff function at time \(jN\Delta t \ (j=0,1,\cdots,J)\) to \(g_{\ell }^j=G(Se^{\mu (\theta)jN\Delta t}e^{\ell \Delta x})\). As discussed in the main text, we can now calculate the American option price using a discrete cosine transform.B Appendix-B:Accuracy of the DCT Method for Pricing American OptionsIn this section, we estimate the numerical accuracy of the DCT method proposed in the previous section. In this article, we construct an approximation tree based on the Markov chain, which has two different sources of approximation error. The first is because of the constraint imposed on the range of the payoff function in our algorithm, while the second is due to the characteristic function. Although the range of the tree is constrained, this constraint is not included in the calculation of the characteristic function. With this in mind, we proceed to estimate the numerical error between the results obtained by our proposed methods and those derived from the original tree. We fix the integer N at \(N=1\) to evaluate the price of American options with expiration date \(T=J \Delta t\). We denote the price of the underlying asset at time \(j \Delta t\) by \(S_{j \Delta t}\). We assume that the discrete-time stochastic process \(\{ S_{j \Delta t} \}_j\) evolves on the tree. The transition probability is \(p_{k,\ell }={\mathbb P}\left[S_{(j+1) \Delta t}=s_{\ell }|S_{j \Delta t}=s_k \right]\), as described in the previous section. We also introduce a new probability, \(p^m_{k,\ell }\), as \(p^m_{k,\ell }={\mathbb P}\left[S_{(j+m) \Delta t}=s_{\ell }|S_{j \Delta t}=s_k \right]\). Remark B.1 For the sequence \(\{ {\alpha }_n \}_{n=a}^{b}\), we define another sequence, \(\{ {\tilde{\alpha }}_n \}_{n=-\infty }^{\infty }\), which is defined as \({\tilde{\alpha }}_n={\alpha }_n\), if an integer \(n=2M k +r\) satisfies \(r=\ell\) or \(r=2b+1-\ell\) using \(a \le \ell \le b\). The American option price in the binomial tree model at time \(j \Delta t\) is represented by \(v^j_{\ell }\) when the underlying price is \(s_{\ell }=Se^{\ell \sqrt{\Delta t}}\). This is computed using the DCT approach. The numerical result is denoted by \(w^j_{\ell }\). In this section, we estimate the numerical error, \(\epsilon =v_{0}^{0}-w_{0}^{0}\). We fix an integer \(\ell \in [a,b]\) and estimate the error in the price of options without early exercise,$$\epsilon _{\ell }^{J-1}=c_{\ell }^{J-1}-d_{\ell }^{J-1}:=\zeta _1^{\ell }(J-1)+\zeta _2^{\ell }(J-1)\;,$$where \(\{c_{\ell }^{J-1}\}_{\ell =a}^b\) is the price of options without early exercise. Furthermore, \(\{d_{\ell }^{J-1}\}_{\ell =a}^b\) is the numerical price of Bermudan options without early exercise computed using the DCT method. The first term, \(\zeta _1^{\ell }(J-1)\), is the error caused by ignoring the outside of the payoff for the interval [a,b]. This term arises from the first approximation of Eq.2.4. The second term, \(\zeta _2^{\ell }(J-1)\), is an error caused by using the outside of [a,b] in calculating the characteristic function. This term results from the second approximation of Eq.2.4. These quantities are expressed as follows:$$\begin{aligned} \zeta _1^{\ell }(J-1)=e^{-r \Delta t}\left(\sum _{j< a \ or \ b